THE NATIONAL TECHNICAL UNIVERSITY OF UKRAINE
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of Mathematical Physics and Differential Equations

HIGHER MATHEMATICS PART 3.
Series. Theory of the Functions of a Complex Variable.
SYLLABUS

Description of the Discipline

Level of HE	*The first bachelor*
Branch of Knowledge	*13 Mechanical engineering*
Specialty	*131 "Applied Mechanics"*
Educational Program	*Manufacturing Engineering*
Status of the Discipline	*Compulsory.*
Form of Study	*Full-time*
Year and Semester	*Second year, autumn semester*
Number of Credits	*4 credits ECTS (120 hours), including 36 hours of Lectures, 36 hours pf Practical Lessons, 48 hours of Self-study work*
Semester's Control	*Exam/ Midterm test, Individual tasks*
Schedule	*According to the schedule on the university website*
Language	*English*
Information About Course Teacher	*Ganna Zhuravska, Ph.D., Associate Professor* https://mph.kpi.ua/osobovij-sklad.html
Learning Resources	*Learning resources are determined by the course teacher and are given to students in on the first lesson. They consist of links to remote resources in Moodle, Google classroom, information resources in the university library, on the website of the department, etc.*

Course Description

1. Learning Objectives, Tasks and Outcomes of the Discipline

The discipline "Higher Mathematics Part 3. Series. Theory of the Functions of a Complex Variable" is the third credit module belongs to the discipline "Higher Mathematics", which is included in the cycle of natural-scientific trainings for bachelors of the specialty 131 "Applied Mechanics".

The purpose of the discipline is studying the basic concepts and methods of the theory of numerical and functional series, functions of a complex variable, operational calculus; developing of the student's capability to use theory for solving typical problems on these topics; application of acquired knowledge, skills and abilities to solve applied problems of mathematics, mechanics, physics and in their daily practice; independent usage and studying of mathematical literature and other information sources.

Objective of the discipline is shaping the following knowledge and skills of students.

According to the matrix of correspondence of program competencies

- *general competencies:*

GC1. Ability to abstract thinking, analysis and synthesis;

- *professional competencies:*

PC1. Ability to analyze materials, structures and processes based on laws, theories and methods of mathematics, natural sciences and applied mechanics.

Students must demonstrate the following learning outcomes.

According to the matrix of correspondence of program learning outcomes in the educational program

- LO1) Ability to choose and apply convenient mathematical methods for solving problems of applied mechanics.
- **Knowledge:** the bases of the theory of series (concept of numerical series, convergence and divergence of a series, absolute and conditional convergence for alternating series, function series, power series, interval of convergence, Taylor series, Fourier series, applications of series); the fundamentals of the theory of functions of a complex variable and an operational calculus (concepts, derivatives, and integrals, Taylor series and Laurent series of functions of a complex variable, Laplace and Fourier transformations, application of the Laplace transformation for solving ordinary differential equations and systems of such equations).
- **Skills:** to study numerical, functional and power series, to use Taylor series and Fourier series, to orientate in the fields of their application; to find derivative and to calculate integrals of a functions of a complex variable, to represent a function in the form of Taylor or Laurent series; to use Laplace transforms for solving ordinary differential equations and systems of such equations.
- **Experience:** thinking logically and flexibly; independently using and studying literature on mathematics; generalizing, setting the goal and choosing ways to solve it; using methods of mathematics in engineering calculations.

2. Prerequisites and Postrequisites of the Discipline (place in the structural and logical scheme of education according to the relevant educational program)

Interdisciplinary connections:

The discipline "Higher Mathematics Part 3. Series. Theory of the Functions of a Complex Variable " is based on the previous two credit modules "Higher Mathematics Part 1. Differential and integral calculus of a function of one variable" and "Higher Mathematics Part 2. Differential and Integral Calculus of the Function of Several Variables. Differential Equations ", which were studied in the previous two semesters. This discipline provides other disciplines of the natural sciences cycle such as Physics, Engineering Graphics, Computer Science, Electrical Engineering and Electronics; must precede and provide the following disciplines in the program of professional and practical training of a specialist: Engineering Mechanics, Strength of Materials, Technology of Structural Materials, Theory of Mechanisms and Machines, Theoretical Mechanics, Mathematical Modeling and other disciplines (according to the structural and logical scheme of the educational program).

3. Content of the Discipline

Part 11. Numerical Series.

Topic 11.1. Basic Concepts and Definitions of Numerical Series. Properties of Numerical Series. Necessary Condition for Convergence of a Series.

Topic 11.2. Comparing Series with Positive Terms. Tests of Convergence.

Topic 11.3. Alternating Series. Leibniz’s Theorem. Absolute and Conditional Convergence.

Part 12. Functional Series.

Topic 12.1. The Concept of a Functional Series. Domain of Convergence. Uniform Convergence of a Functional Series. The Continuity of the Sum of a Functional Series. Integration and Differentiation of Functional Series.

Topic 12.2. Power Series. Interval of Convergence. Properties of Power Series.

Topic 12.3. Taylor's and Maclaurin's Series. Examples of Expansion of Functions in Series.

Topic 12.4. Application of Power Series.

Part 13. Fourier series.

Topic 13.1. The Concept of a Fourier Series. Expansion of Functions in Fourier Series. Sufficient Conditions for Expandability.

Topic 13.2. Fourier Series for Even and Odd Functions. Fourier Series for Periodic Functions.

Topic 13.3. Fourier Series for Functions Defined on an Arbitrary. Application of Fourier Series.

Part 14. Elements of the Theory of Functions of a Complex Variable.

Topic 14.1. Complex Numbers and Operations on Complex Numbers. The Concept of a Function of a Complex Variable. Limits and Continuity.

Topic 14.2. Differentiating the Function of a Complex Variable. Cauchy-Riemann Conditions. Analytic Functions and Their Properties.

Topic 14.3. The Geometric Meaning of the Derivative of a Function of a Complex Variable. The Concept of a Conformal Mapping.

Topic 14.4. An Integral with Respect to a Complex Variable. Definition and Properties. Cauchy's Integral.

Topic 14.5. Power Series on the Complex Plane. Taylor's and Laurent Series. Expansion of an Analytic Function in a Laurent Series.

Topic 14.6. A Classification of the Isolated Singular Points of a Single-Valued Analytic Function. The Residue of an Analytic Function at an Isolated Singularity. Application.

Part 15. Operational calculus.

Topic 15.1. Definition of the Laplace Transformation. Laplace Transformations of Some Elementary Functions.

Topic 15.2. The Main Properties of Laplace Transformation.

Topic 15.3. Determining the Original Function from the Transform. Mellin's formula.

Topic 15.4. Solving Problems for Linear Differential Equations and Systems Linear Differential Equations of by the Operational Method.

Topic 15.5. Fourier Integral and Fourier Transformation. Complex Form of a Fourier series.

4. Informational Support

Readings

1. N.Piscunov Differential and Integral Calculus/ N.Piscunov – Mir Publisher, Moscow, 1966 – 895 p.
2. Ganna Zhuravska Higher Mathematics. Series. Elements of Theory [Electronic resource] / Ganna Zhuravska. – Electronic text data (1 file: 1,3 MB). – Kyiv : Igor Sikorsky Kyiv Polytechnic Institute, 2021. – 67 p.

https://ela.kpi.ua/handle/123456789/41247

3. A.G.Sveshnikov The Theory of Functions of a Complex Variable/ A.G.Sveshnikov, A.N.Tichonov – Mir Publisher, Moscow, 1982 – 344 p. https://archive.org/details/SveshnikovTikhonovTheTheoryOfFunctionsOfAComplexVariable
4. B.A. Fuchs Functions of a Complex Variables and Some of Their Applications/ B.A. Fuchs, B.V. Shabat – Elsevier, 1964 – 458 p.– ISBN 978-0-08-009404-5 https://www.sciencedirect.com/science/book/9780080094045

Educational Content

5. Learning Activities and Teaching Methods

Basic teaching methods:

1. Providing during lectures theoretical material and examples of solving the main practical problems.

2. Solving examples and tasks during practical trainings through active and collective learning, part-search and research methods;

3. Self-studying of students, such as homework to each topic of the course and individual tasks.

4. As an addition to traditional training activities there are proposed such activities as internet mailing, distance and online learning, online testing etc.

5. Assessment is carried out with the help of various control measures: midterm tests, mathematical dictation, classroom activities, homework, individual tasks and final exam.

Lectures

№	The title of the lecture and a list of the main subtopics
1	Basic Concepts and Definitions of Numerical Series. Properties of Numerical Series. Necessary Condition for Convergence of a Series. Comparing Series with Positive Terms. Tests of Convergence.
2	Alternating Series. Leibniz's Theorem. Absolute and Conditional Convergence.
3	The Concept of a Functional Series. Domain of Convergence. Uniform Convergence of a Functional Series. The Continuity of the Sum of a Functional Series. Integration and Differentiation of Functional Series.
4	Power Series. Interval of Convergence. Properties of Power Series.
5	Taylor's and Maclaurin's Series. Examples of Expansion of Functions in Series.
6	Application of Power Series.
7	The Concept of a Fourier Series. Expansion of Functions in Fourier Series. Sufficient Conditions for Expandability.
8	Fourier Series for Even and Odd Functions. Fourier Series for Periodic Functions. Fourier Series for Functions Defined on an Arbitrary. Application of Fourier Series.
9	Complex Numbers and Operations on Complex Numbers. The Concept of a Function of a Complex Variable. Limits and Continuity.

10	Differentiating the Function of a Complex Variable. Cauchy-Riemann Conditions. Analytic Functions and Their Properties.
11	An Integral with Respect to a Complex Variable. Definition and Properties.
12	Cauchy's Integral.
13	Power Series on the Complex Plane. Taylor's and Laurent Series. Expansion of an Analytic Function in a Laurent Series.
14	A Classification of the Isolated Singular Points of a Single-Valued Analytic Function.
15	The Residue of an Analytic Function at an Isolated Singularity. Application.
16	Definition of the Laplace Transformation. Laplace Transformations of Some Elementary Functions. The Main Properties of Laplace Transformation.
17	Determining the Original Function from the Transform. Mellin's formula. Solving Problems for Linear Differential Equations and Systems Linear Differential Equations of by the Operational Method.
18	Fourier Integral and Fourier Transformation. Complex Form of a Fourier series.

Practical Trainings

№	The title of the practical training
1	Numerical Series. Necessary Condition for Convergence of a Series. Comparing Series with Positive Terms.
2	Tests of Convergence: Ratio Test, Root Test, Integral Test.
3	Alternating Series. Leibniz's Theorem. Absolute and Conditional Convergence.
4	Functional Series. Domain of Convergence. Uniform Convergence of a Functional Series.
5	Power Series. Interval of Convergence.
6	Taylor's and Maclaurin's Series. Expansion of Functions in Series.
7	Application of Power Series.
8	Fourier Series. Expansion of Functions in Fourier Series.
9	Test 1.
10	The Concept of a Function of a Complex Variable. Limits and Continuity.
11	Differentiating the Function of a Complex Variable. Cauchy-Riemann Conditions. Analytic Functions and Their Properties.
12	An Integral with Respect to a Complex Variable. Cauchy's Integral.
13	Power Series on the Complex Plane. Taylor's and Laurent Series.
14	A Classification of the Isolated Singular Points of a Single-Valued Analytic Function. The Residue of an Analytic Function at an Isolated Singularity. Application.
15	Test 2.
16	Laplace Transformation. Laplace Transformations of Some Elementary Functions. The Main Properties. Determining the Original Function from the Transform.
17	Solving Problems for Linear Differential Equations and Systems Linear Differential Equations of by the Operational Method.
18	Fourier Integral and Fourier Transformation. (1 hour) Test 3. (1 hour)

6. Self-studying of Students

There are 48 hours of self-study work of students. They include 18 hours for learning lectures and doing homework, 5 hours for preparing to midterm tests, 5 hours for doing individual tasks and 20 hours for preparing for final exam.

Self-study work also carries out studying some topics of the discipline.

Self-study Work

№	The title of topic for the self-study	Hours
Part 14. Elements of the Theory of Functions of a Complex Variable.		
1	Topic 14.3. The Geometric Meaning of the Derivative of a Function of a Complex Variable. The Concept of a Conformal Mapping.	1

Policies and Assessments

7. Policies of the Discipline

Class-meetings are held in auditoriums according to the schedule. In the case of online classes that should be provided by the relevant order of the university, lectures and practical trainings are engaged in online video communications (Zoom, Google-Meet, Skype).

Requirements and the system of assessment are announced to students in the first lesson.

Attendance

Students are expected to attend each class meeting, since during these lessons they study theoretical material and develop the skills needed to complete homeworks, individual tasks and middle tests. There are no penalty points for absence from lectures and practical classes.

Absence During Control

Failure to submit the scheduled time the individual tasks and systematic failure to do homeworks without a valid reason is punishable by penalty points, according to the Rating System. Absence from the middle test, if the reason is not documented, is not give you a chance to do it in another time.

Academic Integrity Policy

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

Norms of Ethical Behavior

Norms of ethical behavior of students and teachers are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute".

Read more: https://kpi.ua/code.

8. Assessment of Learning Outcomes and Rating System

Execution ongoing control and assessment of student's knowledge are made due to classroom activities, middle tests and individual tasks, the purpose of which is to identify the level of assimilation of relevant topics.

The final test is an exam in the end of each semester. The 100-points rating system and the university scale are used to evaluate the results of the training.

The Rating System

1. The student rating of the credit module is calculated from 100 points, 60 of them are the starting scores. Starting scores (during the semester) consists of the points that the student receives for:

1) classroom activities (18 lessons);
2) homework;
3) three midterm tests;
4) individual tasks.

2. Criteria For Scoring Points

2.1. Classroom Activities:

– student has a thorough knowledge and understanding of the content and a high level of competence in the processes and skills - 2 points;

– student has achieved a basic level of competence in the processes and skills - 1 point;

– student has no elementary knowledge and understanding of the content - 0 points.

The maximum of points for classroom activities during the semester is 20.

2.2. Homework

It includes completing tasks at home for each topic covered. Correctly completed and timely homework is assessed at 0.2 points.

The maximum of points for all homework is 0.2 ×15=3 points.

2.3. Midterm Test

There are three Midterm tests (two for 8 points and one for 6 points)

- complete solutions (no less than 90% of the required information) –7-8 points or 5,5-6 points;

- solutions with minor inaccuracies (no less than 75% of the required information) - 6 points or 4,5-5 points;

- mistakes in an solutions (no less than 60% of the required information) - 4,5-5 points or 3,5-4 points;

- no solutions - 0-4 points or 0-3 points.

Midterm test №1	Max points
Topic: "Series" (2 hours)	8
Midterm test №2	
Topic: "Theory of Functions of a Complex Variable" (2 hours)	8
Midterm test №3	
Topic: "Operational calculus" (1 hours)	6

Use of a cell phone or any other communication device is not permitted during midterm tests. Violation of this policy will result in a score of 0 (zero) in the midterm tests.

Make-ups of the midterm test is allowed only in the case of a valid reason.

In the case of online learning, the midterm test is written by students in practical classes using Zoom or Skype platforms (or another, depending on the agreement with the teacher).

Students are sent the card for the test, and they, in two academic hours, must send photocopies of written solution via messenger (Telegram, What's Up etc.) or by e-mail. If the student's work is not sent in time, it is considered that the student was absent from the test, the work is not checked, and student receives 0 points.

2.4. Individual Tasks - 15 points.

- complete solutions – 13,5-15 points;
- solutions with minor inaccuracies - 13-12 points;
- mistakes in an solutions - 9-10 points;
- no solutions - 0-8 points.

Each week of delay of the submission of the individual tasks to examination are provided with penalty (-1) points.

The student must submit the completed individual tasks no later than the last lesson of the semester. In case of violation of this deadline, the student is considered not admitted to the exam of the main session. Student could be allowed to pass exam should submit the individual tasks before the additional session.

In the case of online learning, the individual tasks is checked by sending photocopies of the written work to the teacher's e-mail (or other platform, depending on the agreement with the teacher (Telegram, What's Up etc.)).

3. The requirement to the first midterm assessment (attestation) is to receive at least 11 points and to submit the corresponding part of individual tasks. The requirement to the second midterm assessment (attestation) is to receive at least 22 points and to submit the corresponding part of individual tasks.

4. The allowance requirement to passing the exam is the individual tasks, that must be credited, and no less than 36 points of starting score.

5. At the exam each student have two theoretical questions (8 points each) and three practical ones (8 points each). Total exam score is 40 points.

The criterion for evaluating theoretical questions:

- full answer (not less than 90% of the required information) - 8 points.
- sufficiently complete answer (not less than 75% of the required information, or minor inaccuracies) - 6-7 points.
- incomplete answer (not less than 60% of the required information and some mistakes) - 4-5 points.
- wrong answer - 0 points.

Criteria for evaluating practical tasks:

- complete solution of the task - 8 points.
- solution of the task with minor inaccuracies - 6-7 points.
- the task is solved with some mistakes - 4-5 points.

- the task is not solved - 0 points.

If the exam score is less than 24 points, then the student gets an "unsatisfactory" rating.

Use of books, a cell phone or any other communication device is not permitted during exams. Violation of this policy will result in a score of 0 (zero) in the exam.

6. The sum of the starting score and the exam score (personal score) is transferred into the grade of credit module according to the table:

PERSONAL SCORE	**GRADE**
95-100	Excellent
85-94	Very Good
75-84	Good
65-74	Satisfactory
60-64	Sufficiently
Personal score<60 or the exam score is less than 24 points	Unsatisfactory
Personal score < 36 or the individual tasks is not credited	Not allowed to pass exam

In the case of online learning, according to the teacher's decision, it is possible to set the exam grade by recalculating the starting points on a 100-point scale:

$$R = 60 + \frac{40(R_I - R_D)}{R_C - R_D},$$

where R_I - the amount of points earned by the student during the semester (starting score).

If the student does not agree with that grade, he takes the exam in video mode according to the schedule of the examination session.

Syllabus of the Discipline:

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Approved:

Department of Mathematical Physics and Differential Equations (Protocol № 9 dated July 7, 2022)

Agreed:

Methodical Commission of Institute of Mechanical Engineering (Protocol №11 dated August 29, 2022)